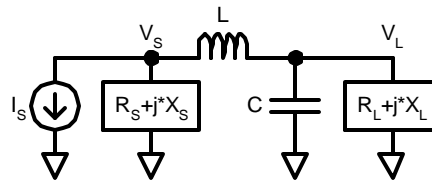


# Lowpass L Matching Network Designer



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## General Impedance Matching

This worksheet discusses in detail the impedance matching with application to the synthesis of a lowpass L-shaped impedance matching network. The design of the matching networks is performed graphically in many textbooks using a smith chart. In this report, the design is performed using closed form expressions.

A few notes on matching networks before we start: Matching networks have two main purposes. The first is to deliver allow the maximum power transfer from a source in series with a source impedance of  $Z_S$  to a load impedance  $Z_L$ . The second is to provide a desired impedance to a circuit to achieve other desired characteristics, such as to filter, to maintain a desired filter transfer characteristics, to an LNA to provide a low noise figure, or to a transmission line to reduce reflections. Here the matching networks are designed for narrowband networks, such as RF transmitters and receivers, but not broadband applications, such as audio applications, cable modems, backplanes, ethernet, etc.

Let's start with the available source power and load power definitions:

$$P_S = \frac{V_{Srms}^2}{R_S} \quad P_L = \frac{V_{Lrms}^2}{R_L} \quad V_{Lrms} = V_{Srms} \cdot \frac{Z_L}{Z_S + Z_L}$$

The power gain of the network is found by dividing the expression for the output power divided by the input power.

$$G_p = \frac{P_L}{P_S} = \left( \frac{Z_L}{Z_S + Z_L} \right)^2 \cdot \frac{R_S}{R_L} = \left[ \frac{R_L^2 + X_L^2}{(R_S + R_L)^2 + (X_L + X_S)^2} \right] \cdot \frac{R_S}{R_L}$$

To optimize the power gain for a given load impedance the derivative of power gain with respect to the source resistance and reactance is set to zero, and the source impedance is solved. This is easiest done in a two step process by first finding the optimal source reactance,  $\frac{d}{dX_S} G_p = 0$ , which gives an optimal source reactance of:

$$X_{Sopt} = -X_L$$

The source resistance can be found similarly with  $\frac{d}{dR_S} G_p = 0$ , giving an optimal source resistance of:

$$R_{Sopt} = R_L$$

This impedance is also said to give zero power reflection. That is all of the power is transferred to the load, and none is reflected back to source. When viewing the power signals as waves it is difficult to view a reflection, when the matching network consists of discrete elements. When studying distributed structures, such as transmission lines, we see the find that zero reflection occurs for true matched line and load impedance (not conjugate). So if you are delivering power from a source through an input matching network, through a transmission line, through an output matching network, and finally to a load, how is that you can provide maximum power delivery to the load with a conjugate match, but have zero reflections? The answer is: "You can't, " because this situation does not occur in practice. In practice in order for a transmission line to have an imaginary characteristic impedance it must be lossy.

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## Impedance Transformation for Power Amplifiers

Careful! Output networks for power amplifiers are more commonly "impedance transformation" networks rather than "impedance matching" networks. The goal of a power amplifier is to efficiently transfer power from a **DC** power supply to a load at an RF frequency; however an impedance matching network provides maximum power transfer from an **RF** source to a load. It turns out the two are the same for output powers less than  $(V_{DD} - V_{DSsat})^2 / 2R_L$ , which is around 100mW for a 50 ohm load and 3V supply. For powers above this, the load is **transformed** to an impedance, seen from the power amplifier, which is much lower than the output impedance of the PA.

- ▢ useful functions and identities
- ▢ Units
- ▢ Constants

## Inputs

$Z_L := 200\text{ohm} - j\cdot 50\text{ohm}$	Load impedance	
$Z_S := 50\text{ohm} + j\cdot 25\text{ohm}$	Source impedance	
$f_{\min} := 900\text{MHz}$	Lower edge of band frequency	
$f_{\max} := 900\text{MHz}$	Upper edge of band frequency	
$I_i := 100\text{mA}_{\text{rms}}$	Input current	
$V_{i\max} := 4\text{V}_{\text{rms}}$	Input voltage constraint	$V_i < V_{i\max}$
$V_{o\max} := 4\text{V}_{\text{rms}}$	Output voltage constraint	$V_o < V_{i\max}$

## Calculations

The L matching network is only designed to provide maximum power transfer at one frequency. In other words it is a narrow band matching network. Given a band of input frequencies, we set design the matching network for the center of the band.

$$f_c := \frac{f_{\min} + f_{\max}}{2} \quad f_c = 900\text{MHz}$$

$$f := f_c \quad \omega_c := 2 \cdot \pi \cdot f_c \quad \omega := 2 \cdot \pi \cdot f$$

The first step in synthesizing an impedance matching network is to deimbed the source and load reactances. Because the matching network consists of an inductor in series with the source, the source impedance is converted into an equivalent series resistance and inductance. The equivalent series inductance and resistance of source impedance are:

$$R_S := \text{Re}(Z_S) \quad R_S = 50\text{ohm}$$

$$L_S := \frac{\text{Im}(Z_S)}{\omega} \quad L_S = 4.421\text{ nH}$$

Deimbedding the load reactance is a little more difficult, because the impedance is in parallel. The calculation is easy to perform by converting the input impedance to an admittance with a complex math processor, such as Mathcad, but is performed here assuming only a real processor is available. The equivalent parallel capacitance and resistance of load impedance are:

$$Q_L := -\frac{\text{Im}(Z_L)}{\text{Re}(Z_L)} \quad Q_L = 0.25$$

$$R_L := \text{Re}(Z_L) \cdot (1 + Q_L^2) \quad R_L = 212.5\text{ohm}$$

$$C_L := \frac{Q_L}{\omega \cdot R_L} \quad C_L = 0.208\text{ pF}$$

With the source and load reactances deimbedded, we can synthesize the matching network using the real part of the source and the load. Our goal here is to size the L and the C such that source and the load sees the conjugate of its impedance looking into the network. A matched network sliced anywhere in the network will have a conjugate impedance looking in either direction.

$$Z_{\text{left}} = \overline{Z_{\text{right}}}$$

The right side of the matching network consists of a parallel resistance and reactance. This can be expressed in a simplified form with the parallel resistance and the Q of the source.

$$Z_{\text{right}} = \frac{1}{\frac{1}{R_p} + \frac{1}{j \cdot B_{\text{right}}}} = \frac{1}{G_p + j \cdot B_{\text{right}}} = \frac{R_p}{(1 + j \cdot Q_{\text{right}})}$$

$$R_{\text{right}} + j \cdot L_{\text{right}}$$

The left side of the matching network consists of a series resistance and reactance. This can be expressed in a simplified form with the series resistance and the Q of the load.

$$\overline{Z}_{\text{left}} = R_{\text{left}} + \frac{1}{j \cdot C_{\text{left}}} = R_S + j \cdot X_{\text{left}} = R_S \cdot (1 - j \cdot Q_{\text{left}})$$

Because the left and right resistances must be equal, and the reactances conjugates of each other, the Q's of the left and right sides must be equal. Setting the left impedance and right conjugate equal to each other, and setting the Q's equal gives the following equality.

$$\frac{R_p}{(1 + j \cdot Q)} = R_S \cdot (1 - j \cdot Q)$$

Solving this expression gives a general expression for the Q of matching network. Note the Q value is fixed by the source and load impedance for the L matching network. For higher order networks, and for equal source and load resistance, the Q of the network may be chosen.

$$Q := \sqrt{\frac{R_L}{R_S} - 1} \quad Q = 1.803$$

This Q will serve as a lower bound for pi and T matching networks, and as an upper bound for cascaded L matching networks. Note the difference between network Q and component Q. Network Q is usually on the order of zero to three, to reduce sensitivity of the network. Component Q is much higher, typically 10 to 100, to

Given the Q of the left and right half sides of the network, and the effective source and load impedances, we can find the matching network inductance and capacitance without source inductance deembedded

$$L_T := \frac{Q \cdot R_S}{\omega} \quad L_T = 15.94 \text{ nH}$$

$$C_T := \frac{Q}{\omega \cdot R_L} \quad C_T = 1.5 \text{ pF}$$

The final impedance network is found by subtracting the source inductance from the matching inductance and the load capacitance from the matching capacitance.

$$L := L_T - L_S \quad \boxed{L = 11.519 \text{ nH}}$$

$$C := C_T - C_L \quad \boxed{C = 1.292 \text{ pF}}$$

This optimal power matching network corresponds to an input and output rms voltage swing of

$$V_{i\_I_{\text{opt}}}(\omega) := \frac{Z_S \cdot \left[ L \cdot j \cdot \omega + \frac{Z_L \cdot \frac{1}{j \cdot \omega \cdot C}}{\left( Z_L + \frac{1}{j \cdot \omega \cdot C} \right)} \right]}{Z_S + L \cdot j \cdot \omega + \frac{Z_L \cdot \frac{1}{j \cdot \omega \cdot C}}{\left( Z_L + \frac{1}{j \cdot \omega \cdot C} \right)}} \quad |V_{i\_I_{\text{opt}}}(\omega_c)| I_i = 3.125 \text{ V}_{\text{rms}}$$

$$V_{o\_I_{\text{opt}}}(\omega) := \frac{Z_S}{Z_S + L \cdot j \cdot \omega + \frac{Z_L \cdot \frac{1}{j \cdot \omega \cdot C}}{\left( Z_L + \frac{1}{j \cdot \omega \cdot C} \right)}} \cdot \frac{Z_L \cdot \frac{1}{j \cdot \omega \cdot C}}{Z_L + \frac{1}{j \cdot \omega \cdot C}} \quad |V_{o\_I_{\text{opt}}}(\omega_c)| I_i = 5.762 \text{ V}_{\text{rms}}$$

## Outputs

$$C = 1.292 \text{ pF}$$

$$L = 11.519 \text{ nH}$$

Load Capacitance (in parallel with load)  
Source Inductance (in series with source)

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## Functions

Here's a function to calculate the matching network elements. It is a duplicate of procedure above. It is useful for performing sweeps or references from other routines.

$$\text{LC}(Z_S, Z_L, f) := \left\{ \begin{array}{l}
 \omega \leftarrow 2 \cdot \pi \cdot f \\
 Q_L \leftarrow \frac{\text{Im}(Z_L)}{\text{Re}(Z_L)} \\
 R_S \leftarrow \text{Re}(Z_S) \\
 L_S \leftarrow \frac{\text{Im}(Z_S)}{\omega} \\
 R_L \leftarrow \text{Re}(Z_L) \cdot (1 + Q_L^2) \\
 C_L \leftarrow \frac{Q_L}{\omega \cdot R_L} \\
 Q \leftarrow \sqrt{\frac{R_L}{R_S} - 1} \\
 L_T \leftarrow \frac{Q \cdot R_S}{\omega} \\
 C_T \leftarrow \frac{Q}{\omega \cdot R_L} \\
 L \leftarrow L_T - L_S \\
 C \leftarrow C_T - C_L \\
 \left( \begin{array}{c}
 \frac{L}{H} \\
 \frac{C}{F}
 \end{array} \right)
 \end{array} \right.$$

---

## Example Calculation

$$\begin{aligned}
 x &:= \text{LC}(Z_S, Z_L, f) \\
 L &:= x_1 \cdot H & L &= 11.519 \text{ nH} \\
 C &:= x_2 \cdot F & C &= 1.292 \text{ pF}
 \end{aligned}$$

As a check of the matching network, we calculate the input and output impedance of the network to see if they are the conjugate of the source and the load.

$$\begin{aligned}
 Z_i &:= j \cdot \omega \cdot L + Z_L \parallel \left( \frac{1}{j \cdot \omega \cdot C} \right) & Z_i &= 50 - 25i \text{ ohm} & Z_S &= 50 + 25i \Omega \\
 Z_o &:= (Z_S + j \cdot \omega \cdot L) \parallel \left( \frac{1}{j \cdot \omega \cdot C} \right) & Z_o &= 200 + 50i \text{ ohm} & Z_L &= 200 - 50i \Omega
 \end{aligned}$$

## S Parameters

S-parameter is an abbreviation for scattering parameters. S-parameters are a measure of the power gain of a network. The term scattering comes from the concept of a cue ball scattering other balls as it transfers power to them.  $S_{ij}$  is the measure of power gain from port  $j$  to port  $i$ . Specifically, the square root of power gain. For example,  $S_{21}$ , the most useful S-parameter, is the measure of the ratio of output power to the available input power. This is an important sentence. This sentence is used to perform hand calculations. The output power is easy to explain, it is  $V_{Orms}^2/R_L$ . The available input power is trickier to explain. Available input power is the maximum power that can be delivered from a source. For a source impedance of  $R_S$ , and a source voltage of  $V_{Srms}$ ; the available input power is  $(V_{Srms}/2)^2/R_S$ . Why, because maximum power is delivered is to a resistance of  $R_S$ . Thus a voltage division of two for the voltage, when delivering maximum available power. Thus the power gain of a network,  $S_{21}$ , is found by dividing the two powers  $2 \cdot V_o/V_s \cdot \sqrt{R_S/R_L}$ .

S-parameters are most conveniently calculated by converting an impedance or admittance matrix into a S-parameter matrix. So we start by finding the admittance (Y) parameters for the lowpass L matching network. Admittance (Y) parameters are found by driving a port with a voltage source, shorting the rest, and measuring the current at all the ports.

$$Y(\omega) := \begin{pmatrix} \frac{1}{j \cdot \omega \cdot L} & -\frac{1}{j \cdot \omega \cdot L} \\ -\frac{1}{j \cdot \omega \cdot L} & j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L} \end{pmatrix}$$

Y parameters are converted to a S parameters with a common impedance on all ports using this expression from "Microwave Engineering" by Pozar.

$$Y2S(Y, Y_0) := \begin{cases} I \leftarrow Y_0 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ (I - Y) \cdot (I + Y)^{-1} \end{cases}$$

This scattering parameter conversion routine (to convert to arbitrary source and load impedances) is given "Applied RF Techniques I" lecture notes, but is incorrect. A correct version of the routine is found on page 31 of "Microwave Amplifiers and Oscillators," by Christian Gentili.

$$S_{conv}(S, Z_{Sbegin}, Z_{Send}, Z_{Lbegin}, Z_{Lend}) := \begin{cases} \Gamma_S \leftarrow \frac{Z_{Send} - Z_{Sbegin}}{Z_{Send} + Z_{Sbegin}} \\ \Gamma_L \leftarrow \frac{Z_{Lend} - Z_{Lbegin}}{Z_{Lend} + Z_{Lbegin}} \\ D \leftarrow (1 - \Gamma_S \cdot S_{1,1}) \cdot (1 - \Gamma_L \cdot S_{2,2}) - \Gamma_S \cdot \Gamma_L \cdot S_{1,2} \cdot S_{2,1} \\ A_1 \leftarrow \frac{1 - \overline{\Gamma_S}}{|1 - \Gamma_S|} \cdot \sqrt{1 - (|\Gamma_S|)^2} \\ A_2 \leftarrow \frac{1 - \overline{\Gamma_L}}{|1 - \Gamma_L|} \cdot \sqrt{1 - (|\Gamma_L|)^2} \\ \left[ \frac{\overline{A_1}}{A_1} \cdot \frac{(1 - \Gamma_L \cdot S_{2,2}) \cdot (S_{1,1} - \overline{\Gamma_S}) + \Gamma_L \cdot S_{1,2} \cdot S_{2,1}}{D} \quad \frac{\overline{A_2}}{A_1} \cdot \frac{S_{1,2} \cdot [1 - (|\Gamma_S|)]}{D} \right. \\ \left. \frac{\overline{A_1}}{A_2} \cdot S_{2,1} \cdot \frac{1 - (|\Gamma_L|)^2}{D} \quad \frac{\overline{A_2}}{A_2} \cdot \frac{(1 - \Gamma_S \cdot S_{1,1}) \cdot (S_{2,2} - \overline{\Gamma_L})}{D} \right] \end{cases}$$

$$Z_0 := 50\Omega$$

$$S_{50}(\omega) := Y2S\left(Y(\omega), \frac{1}{Z_0}\right)$$

$$S(\omega) := S_{\text{conv}}(S_{50}(\omega), Z_0, Z_S, Z_0, Z_L)$$

## Reference Impedance for S-Parameters

S-parameters of ideal matching network with actual load and source impedances.

50 Ohm S-parameters of ideal matching network.

Plots of lossy and ideal S-parameters of the matching network versus frequency. Be careful when using these plots, as they do not reflect the change in source and load impedance vs. frequency (for example if driver output impedance is capacitive).

$$\text{numi} := 200$$

$$i := 1.. \text{numi}$$

$$f_{\text{start}} := \frac{f}{5}$$

$$f_{\text{stop}} := f \cdot 50$$

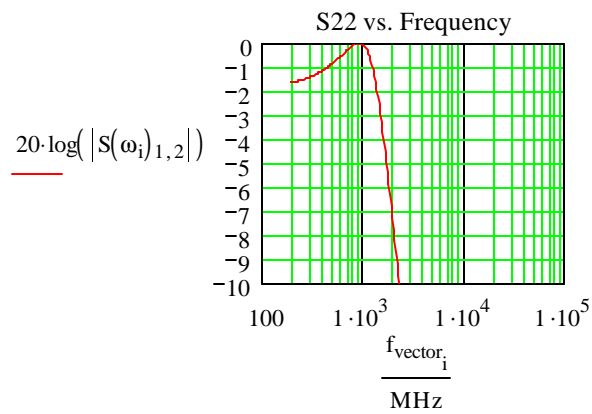
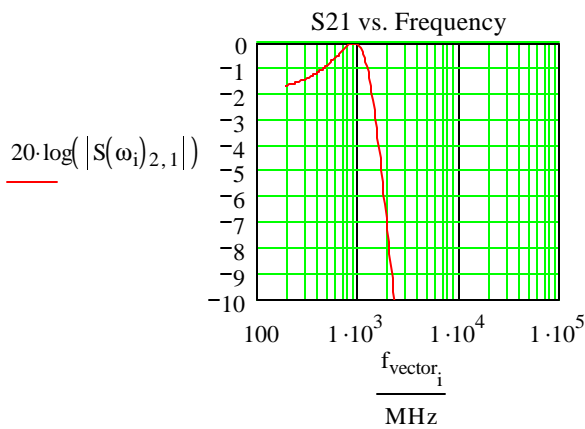
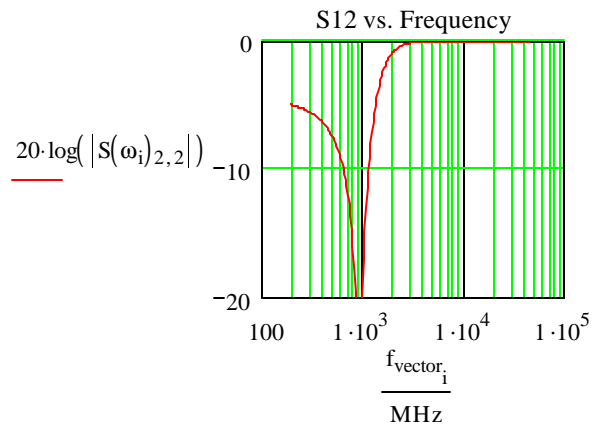
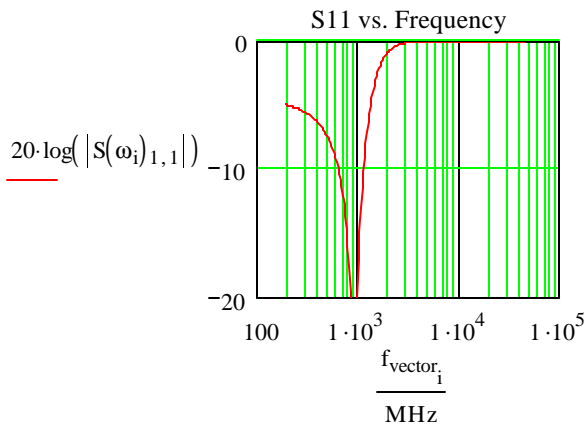
Number of points in frequency plot

Starting and stopping frequencies for plot

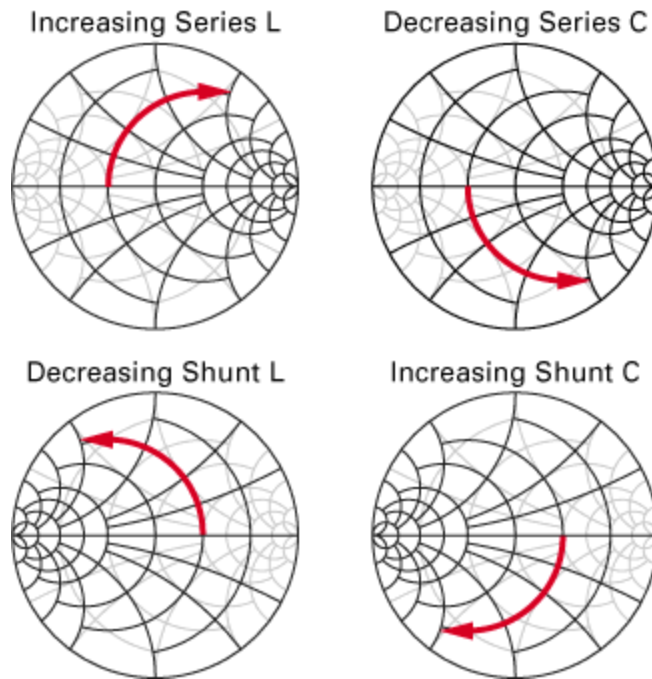
$$f_{\text{vector}_i} := f_{\text{start}} \cdot \left(\frac{f_{\text{stop}}}{f_{\text{start}}}\right)^{\frac{i}{\text{numi}}}$$

$$\omega_i := 2 \cdot \pi \cdot f_{\text{vector}_i}$$

$$s_i := j \cdot \omega_i$$



## Impedance Matching with a Smith Chart



I am personally not a big fan of smith charts, but they have some historical value, are commonly used in network analyzers, and they allow you to plot infinite change in impedance vs. infinite frequency in a finite area. Here we describe a common older method for graphically matching a complex source impedance to a complex load impedance using a Smith chart.

**Step 1:** The source and load impedances are converted to reflection coefficients and placed on the smith chart. One of the two impedances is conjugated, so that a conjugate match is achieved. Here we arbitrarily choose the source to be the conjugated impedance.

$$\Gamma_S := \frac{Z_S - Z_0}{Z_S + Z_0} \quad |\Gamma_S| = 0.243 \quad \angle \Gamma_S = 75.964^\circ \quad \Gamma_L := \frac{\overline{Z_L} - Z_0}{Z_L + Z_0} \quad |\Gamma_L| = 0.62 \quad \angle \Gamma_L = 7.125^\circ$$

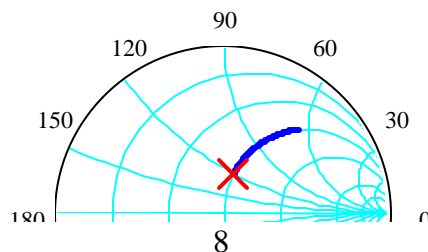
**Step 2:** Using a protractor, with an origin at the left side of the chart, a circle is drawn which includes the source point. This circuit is represents the sweeping of inductance from an zero series inductance to the proper value for matching.

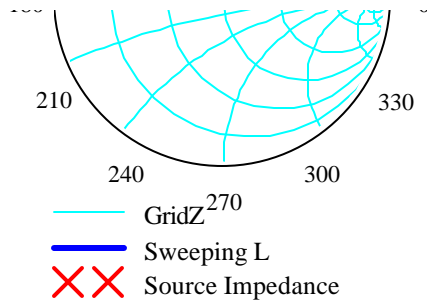
$$L_{\text{vector}_i} := \frac{i}{\text{num}_i} \cdot L$$

$$Z_{\text{left}_i} := Z_S + j \cdot \omega_c \cdot L_{\text{vector}_i}$$

$$\Gamma_{\text{left}_i} := \frac{Z_{\text{left}_i} - Z_0}{Z_{\text{left}_i} + Z_0}$$

▣ Grid

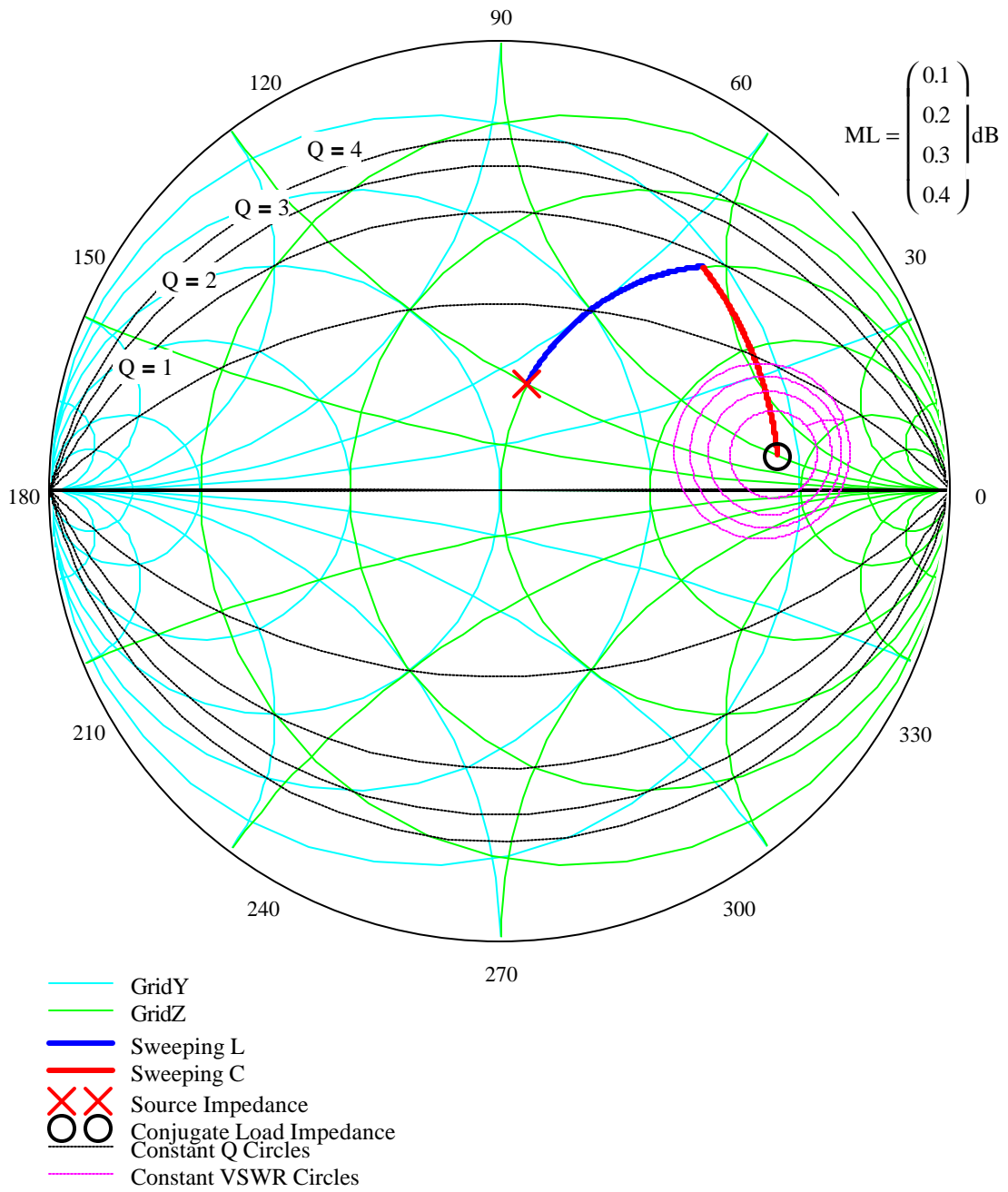




**Step 3:** Using a protractor, with an origin at the right side of the chart, a circle is drawn which includes the load point. This circuit is represents the sweeping of inductance from an zero parallel capacitance to the proper value for matching.

$$Z_{\text{right}_i} := (Z_S + j \cdot \omega_c \cdot L) \parallel \left( \frac{1}{j \cdot \omega_c \cdot C_{\text{vector}_i}} \right) \quad C_{\text{vector}_i} := \frac{C \cdot i}{\text{num}_i} \quad \Gamma_{\text{right}_i} := \frac{Z_{\text{right}_i} - Z_0}{Z_{\text{right}_i} + Z_0}$$

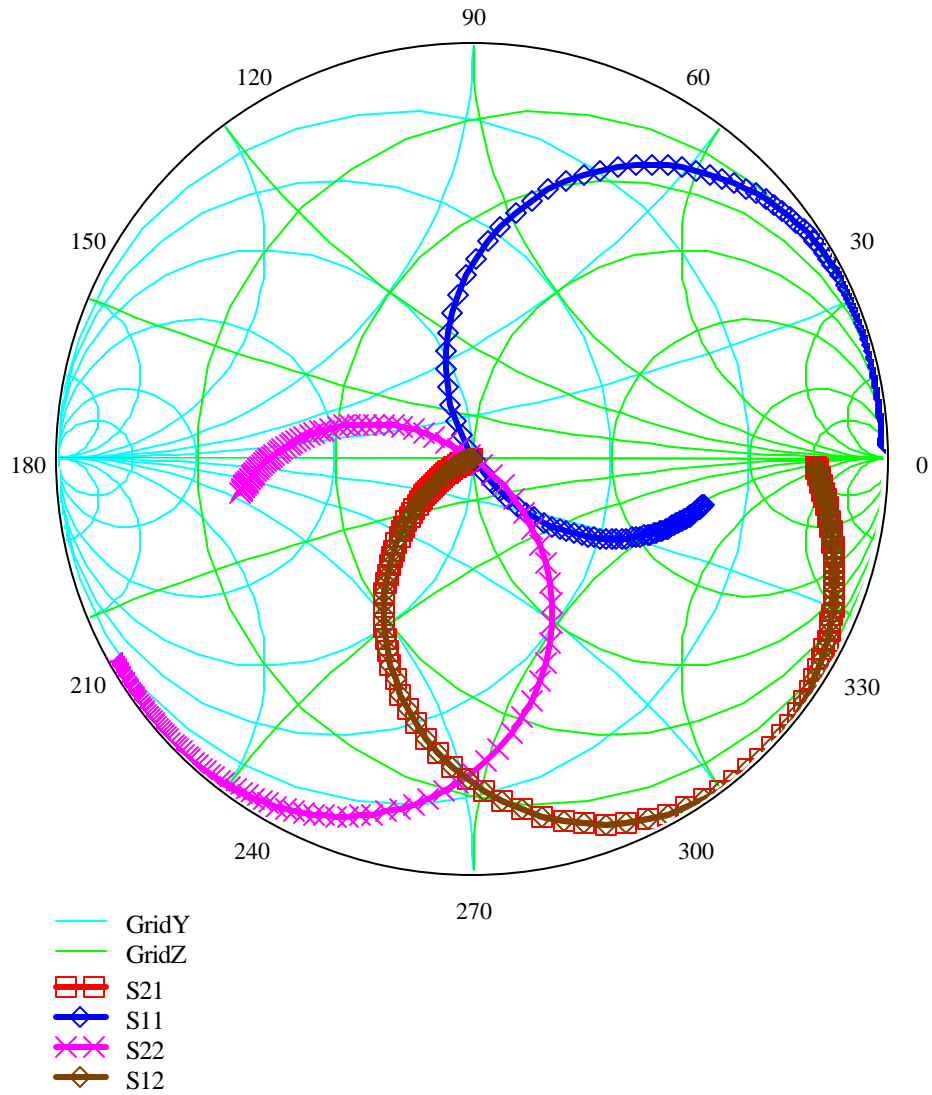
▣ Q and VSWR contours



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## S-Parameters on Smith Chart Plots

Here is a plot of the S parameters for the highpass L matching network vs. frequency on the Smith chart. Ugh, I find this useless, unless you narrow the bandwidth just to the frequency of interest, which hides the filtering properties.



## Sensitivity Analysis

All elements of a real circuit have process variations, which affects the performance of the circuit. The following calculations calculate loss due to these process variations. The calculation is initially performed with purely resistive source and load impedances. For purely passive networks with small variations, the  $S_{11}$  and  $S_{22}$  sensitivities are the same, as are the  $S_{21}$  and  $S_{12}$  sensitivities. Two first order the sensitivity of  $S_{21}$  and  $S_{11}$  to process and center frequencies is zero.

$$Q = \sqrt{\frac{R_L}{R_S} - 1}$$

$$L = \frac{Q \cdot R_S}{\omega}$$

$$C = \frac{Q}{\omega \cdot R_L}$$

$$\Delta\omega_\omega := 0$$

$$\Delta R_{S\_RS} := 0$$

$$\Delta C\_C := 0$$

$$\Delta L\_L := 0$$

$$\Delta R_{L\_RL} := 0$$

$$\omega_c = \omega_c \cdot (1 + \Delta\omega_\omega)$$

$$L = L \cdot (1 + \Delta L\_L) \quad C = C \cdot (1 + \Delta C\_C)$$

$$\operatorname{Re}(Z_L) = \operatorname{Re}(Z_L) \cdot (1 + \Delta R_{L\_RL})$$

$$\operatorname{Re}(Z_S) = \operatorname{Re}(Z_S) \cdot (1 + \Delta R_{S\_RS})$$

### ***S11 as a Function of Mismatch***

$$Z_{in} = j \cdot \omega \cdot L + \frac{R_L}{1 + R_L \cdot C \cdot \omega \cdot j} \quad Z_{out} = \frac{(R_S + j \cdot \omega \cdot L)}{1 + j \cdot \omega \cdot C \cdot (R_S + j \cdot \omega \cdot L)}$$

$$S_{11} = \frac{Z_{in} - R_S}{Z_{in} + R_S} \quad R = R \cdot (1 + \Delta R\_R)$$

$$S_{11} = \frac{(j \cdot \omega \cdot L - R_S) \cdot (1 + R_L \cdot C \cdot \omega \cdot j) + R_L}{(j \cdot \omega \cdot L + R_S) \cdot (1 + R_L \cdot C \cdot \omega \cdot j) + R_L} \quad L = L \cdot (1 + \Delta L\_L)$$

$$C = C \cdot (1 + \Delta C\_C)$$

$$S_{11} = \frac{\left[ \left[ \frac{R_L}{R_S} \cdot (1 - \omega^2 \cdot L \cdot C) - 1 \right]^2 + \left( R_L \cdot C - \frac{L}{R_S} \right)^2 \cdot \omega^2 \right]}{\left[ \left[ \frac{R_L}{R_S} \cdot (1 - \omega^2 \cdot L \cdot C) + 1 \right]^2 + \left( R_L \cdot C + \frac{L}{R_S} \right)^2 \cdot \omega^2 \right]}$$

$$S_{11} = \left( \frac{\Delta R\_R}{2} \right)^2 \quad \text{due to RL variation}$$

$$S_{11} = \left( \frac{\Delta R\_R}{2} \right)^2 \quad \text{due to RS variation}$$

$$S_{11} = \left( Q \cdot \frac{\Delta L\_L}{2} \right)^2 \quad \text{due to L variation}$$

$$S_{11} = \left( Q \cdot \frac{\Delta C\_C}{2} \right)^2 \quad \text{due to C variation}$$

$$S_{11} = \left( \frac{\Delta R\_R}{2} \right)^2 + \left( \frac{\Delta R\_R}{2} \right)^2 + \left( Q \cdot \frac{\Delta L\_L}{2} \right)^2 + \left( Q \cdot \frac{\Delta C\_C}{2} \right)^2$$

## S21 as a Function of Mismatch

$$Z_{in} = j\omega \cdot L + \frac{R_L}{1 + R_L \cdot C \cdot \omega \cdot j} \quad Z_{out} = \frac{(R_S + j\omega \cdot L)}{1 + j\omega \cdot C \cdot (R_S + j\omega \cdot L)}$$

$$Z_{in} = \frac{j\omega \cdot L \cdot (1 + R_L \cdot C \cdot \omega \cdot j) + R_L}{1 + R_L \cdot C \cdot \omega \cdot j}$$

$$Z_{in} = \frac{R_L \cdot \left(1 - \omega^2 \cdot L \cdot C + j \cdot \frac{\omega \cdot L}{R_L}\right)}{1 + R_L \cdot C \cdot \omega \cdot j} \quad Z_{out} = \frac{R_S \cdot \left(1 + j \cdot \frac{\omega \cdot L}{R_S}\right)}{1 - \omega^2 \cdot L \cdot C + R_S \cdot C \cdot \omega \cdot j}$$

$$\frac{V_2}{V_S} = \frac{Z_{in} - j\omega \cdot L}{Z_{in} + R_S} \quad \frac{V_1}{V_S} = \frac{Z_{in}}{Z_{in} + R_S} \quad \frac{V_2}{V_1} = \frac{1}{1 - \omega^2 \cdot L \cdot C + j \cdot \frac{\omega \cdot L}{R_L}}$$

$$S_{21} = \frac{V_2}{V_1} \cdot \frac{2}{1 + \frac{R_S}{Z_{in}}} \cdot \sqrt{\frac{R_S}{R_L}}$$

$$S_{21} = \frac{2}{\frac{R_S}{R_L} + 1 - \omega^2 \cdot L \cdot C + \left(R_S \cdot C \cdot \omega + \frac{\omega \cdot L}{R_L}\right) \cdot j} \cdot \sqrt{\frac{R_S}{R_L}}$$

$$S_{21} = \frac{4}{\left(\frac{R_S}{R_L} + 1 - \omega^2 \cdot L \cdot C\right)^2 + \left(R_S \cdot C \cdot \omega + \frac{\omega \cdot L}{R_L}\right)^2} \cdot \frac{R_S}{R_L} \quad \begin{aligned} R &= R \cdot (1 + \Delta R_R) \\ L &= L \cdot (1 + \Delta L_L) \\ C &= C \cdot (1 + \Delta C_C) \end{aligned}$$

$$S_{21} = (1 + \Delta R_R) \cdot \left(1 - \frac{\Delta R_R}{2}\right)^2$$

$$S_{21} = 1 - \frac{3}{4} \cdot \Delta R_R^2 \quad \text{due to RS process variations}$$

$$S_{21} = 1 - \frac{3}{4} \cdot \Delta R_R^2 \quad \text{due to RL process variations}$$

$$S_{21} = 1 + \frac{1}{4} \cdot Q^2 \cdot \Delta L_L^2 \quad \text{due to L process variations}$$

$$S_{21} = 1 - \frac{1}{4} \cdot Q^2 \cdot \Delta C_C^2 \quad \text{due to C process variations}$$

$$S_{21} = 1 \quad \text{due to frequency variations}$$

$$S_{21} = 1 + \left[ \frac{3}{4} \cdot (\sigma_{\Delta R_{RL}}^2 + \sigma_{\Delta R_{RS}}^2) + \frac{1}{4} \cdot Q^2 \cdot (\sigma_{\Delta C_C}^2 + \sigma_{\Delta L_L}^2) \right]$$

First order sensitivity to process variations is zero

## Amplitude Constrained Impedance Transformation

Often a matching network of this nature is designed with a given input power and has constraints on either the input node or the output node voltage swings. This situation is present in every receiver typically at internal interfaces, such as between an LNA and a mixer, between a VCO and the mixer, and between intermediate stages of a power amplifier chain. These voltage limitations become more evident with process advances, where impedances become high relative to 50 ohms, and supply voltages are reduced to prevent device damage.

Given a constrained input swing, the output swing should be maximized.

Assuming the matching capacitance is lumped into the load reactance, the optimum inductance is found, when it resonates with the load reactance

$$|V_o| = |V_i| \cdot \frac{\sqrt{R_{Leff}^2 + X_{Leff}^2}}{\sqrt{(\omega \cdot L + X_{Leff})^2 + R_{Leff}^2}} \quad L = \frac{-\text{Im}(Z_{Leff})}{\omega} \quad Z_{Leff} = R_{Leff} + j \cdot X_{Leff} = \frac{1}{j \cdot C \cdot \omega} \cdot Z_L = \frac{1}{j \cdot C \cdot \omega} + Z_L$$

Now the input amplitude is

$$|V_{imax}| = |I_i| \cdot \left| \frac{R_{Leff} \cdot (R_S + j \cdot X_S)}{R_{Leff} + R_S + j \cdot X_S} \right| = \frac{|I_i| \cdot R_{Leff} \cdot \sqrt{R_S^2 + X_S^2}}{\sqrt{(R_{Leff} + R_S)^2 + X_S^2}}$$

Solving for  $R_{Leff}$  we get

$$R_{Leff}^2 + 2 \cdot X \cdot R_{Leff} \cdot R_S + X \cdot Z_S^2 = 0 \quad \text{where} \quad X := \frac{(|V_{imax}|)^2}{(|I_i|)^2 \cdot (|Z_S|)^2} \quad X = -1.049$$

$$\frac{(|V_{imax}|)^2}{(|I_i|)^2 \cdot (|Z_S|)^2} - 1$$

$$R_{Leff} := -X \cdot \text{Re}(Z_S) \cdot \left[ 1 + \sqrt{1 - \frac{(|Z_S|)^2}{X \cdot \text{Re}(Z_S)^2}} \right] \quad R_{Leff} = 130.116 \Omega$$

With  $R_{Leff}$  known we can solve for the required matching capacitance

$$R_{Leff} = \frac{\text{Re}(Z_L) \cdot (1 - \text{Im}(Z_L) \cdot \omega \cdot C) + \text{Im}(Z_L) \cdot (R_L \cdot \omega \cdot C)}{(1 - \text{Im}(Z_L) \cdot \omega \cdot C)^2 + (\text{Re}(Z_L) \cdot \omega \cdot C)^2}$$

$$C := \frac{\text{Im}(Z_L)}{\omega_c \cdot (|Z_L|)^2} \cdot \left[ 1 - \sqrt{1 - \left( \frac{|Z_L|}{\text{Im}(Z_L)} \right)^2 \cdot \left( 1 - \frac{\text{Re}(Z_L)}{R_{Leff}} \right)} \right] \quad \boxed{C = 0.454 \text{ pF}}$$

Putting this capacitance in parallel with load gives the effective load impedance

$$Z_{Leff} := \frac{\frac{1}{j \cdot C \cdot \omega_c} \cdot Z_L}{\frac{1}{j \cdot C \cdot \omega_c} + Z_L} \quad Z_{Leff} = 130.116 - 103.535i \Omega$$

Which is used to find the desired matching inductance

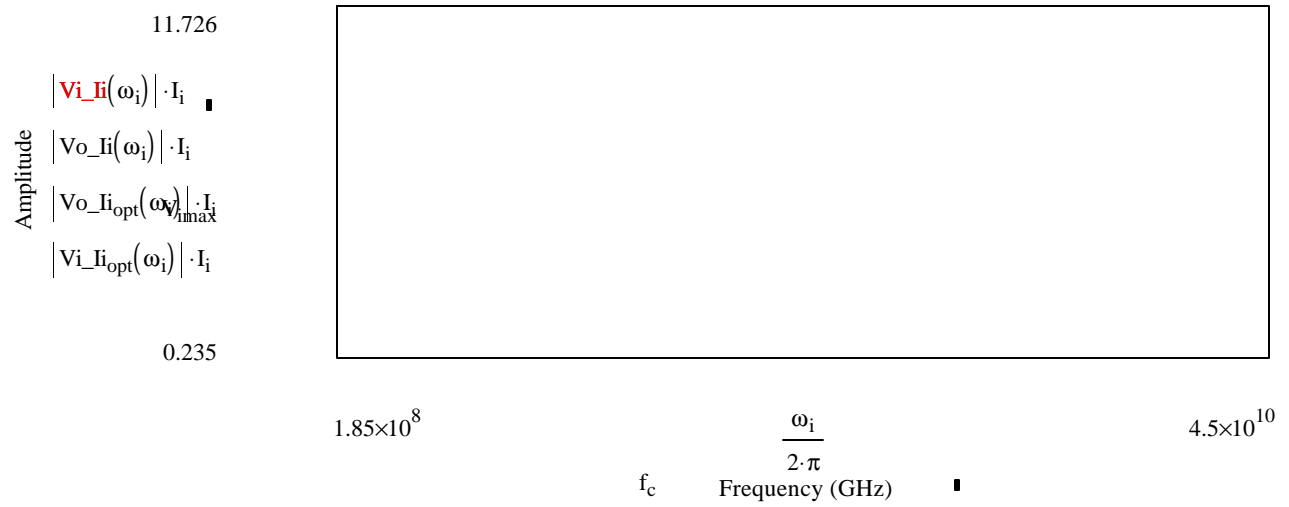
$$L := \frac{-\text{Im}(Z_{Leff})}{\omega_c} \quad \boxed{L = 18.309 \text{ nH}}$$

If the output swing is constrained

Thus the output voltage is constrained to be

where  $G_S$  and  $C_S$  are the parallel equivalent input impedance

Solving for the inductance gives two possible solutions, we choose the one with real inductance values:



## Layout Notes for the Lowpass L Matching Network

The figure at the beginning of this report shows a single inductor placed across the source impedance use to resonate out it's reactance. In real life, there is no such thing as a two-terminal inductance, as inductance is a measure of reactance around a loop. In the figure, the two-terminal inductance represents a partial inductance. The word "partial" is used, because the inductance is only part of a loop. So where is the rest of the loop?

In this circuit there are actually two loops (assuming the source current and resistance are a single lumped element): A source loop and a load loop. In the absence of the matching inductor a single loop exists between the source and load. The matching inductor acts as a shunt in the middle of the loop. The number of loops a circuit has can be easily seen, and is the same number as the number of equations for circuit analysis using KVL. The number of inductors in a circuit is equal to the number of loops, with mutual inductances between every inductance and every other inductance. In a simplified form, the mutual inductances between touching loops may be omitted. Thus the complete matching network is shown in the following figure. Note that the ubiquitous "ground" terminal is placed on only one node as in real life only one point in space can truly be called ground, and in the end it doesn't matter which point that is. The partial inductances are distributed around the entire loop, so it is inappropriate to think of the source ground and the load ground as the same point.

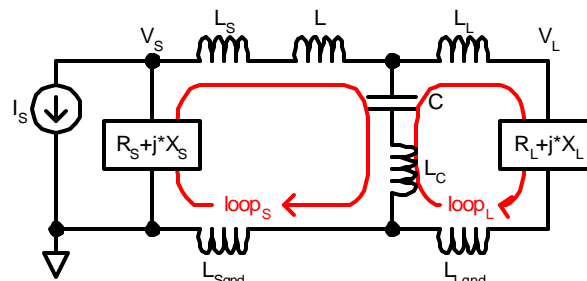


Fig. 1: Lowpass L matching network with all partial inductances.

When designing a matching network it is important to add the parasitic inductances of the source and load loop to the source and load impedances. It is extremely common to design an RF circuit and then find out in testing that the power is matched for a lower frequency, because parasitic inductances were neglected.

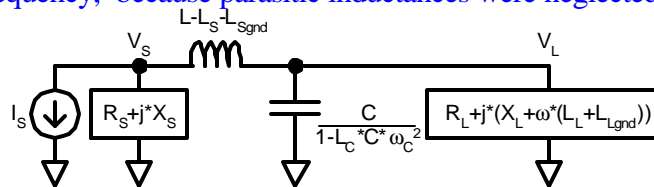


Fig. 1: Lowpass L matching network

These parasitic inductances can be minimized by routing the signal path from the matching network to the load very close to the ground return path from the load. The same is true for the routing of the source to the matching network.

Also to be included in the matching network design is the deembedding of the parasitic capacitances of the matching elements themselves.

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