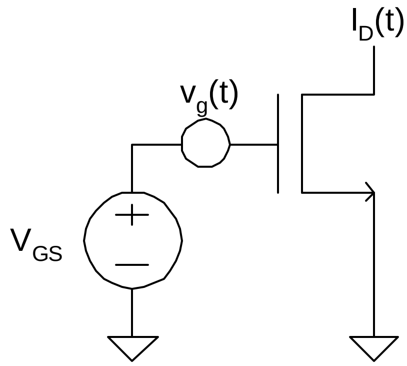
Noise Figure Increase due to Mixing of Bias Noise with Jammer



▸ useful functions and identities
▸ Units
▸ Constants

Table of Contents

Introduction

A simple example of the current noise output from a MOSFET driven in saturation with a large sinusoid around a DC bias. The analysis will be extended and used to analyze and design both bipolar and MOS oscillators.

Model Parameters

$\mu := 300 \cdot \frac{cm^2}{V \cdot sec}$ Device mobility under bias

$t_{OX} := 10 \cdot nm$ Gate oxide thickness

$\varepsilon_r := 3.9$ Relative permittivity of silicon dioxide

$\varepsilon_0 := 8.8542 \cdot 10^{-12} \frac{F}{m}$ Permittivity of free space

$C_{OX} := \frac{\varepsilon_r \cdot \varepsilon_0}{t_{OX}}$ $C_{OX} = 3.453 \frac{fF}{\mu m^2}$

$\mu \cdot C_{OX} = 103.594 \frac{\mu A}{V^2}$

$V_T := 0.7V$ Gate threshold voltage

$\gamma := \frac{2}{3}$ Noise coefficient

$T := (273 + 27)K$ Operating Temperature

$k := 1.3806 \cdot 10^{-23} \frac{V^2}{ohm \cdot Hz \cdot K}$ Boltman's constant

Inputs

$V_{GS} := 1.5V$ DC gate to source bias voltage

$f_0 := 1GHz$ Frequency of input sine wave

$N := 100$ Number of points in time vector

$A := 200 \cdot mV$ Amplitude of signal swing at gate

Calculations

$$t := \frac{2}{f_0 \cdot N}, \frac{4}{f_0 \cdot N} .. \frac{2}{f_0}$$ Time vector

$$v_g(t) := A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$ Large input signal

$$I_D(t) := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T + v_g(t))^2$$ Large signal current in the device.

$$g_m(t) := \sqrt{2 \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot I_D(t)}$$ Time varying small signal transconductance

$$g_m(t) := \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot (V_{GS} - V_T - v_g(t))$$ transconductance written another way

$$v_n(t) := 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{g_m(t)}$$ Input referred device thermal noise

$$I_{spn}(t) := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T + v_g(t) + v_n(t))^2$$ Current of signal plus noise

Expansion of current noise:

$$I_{spn}(t) := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \left[V_{GS} - V_T + A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{\mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot (V_{GS} - V_T + A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t))^2} \right]^2$$

$$I_0 := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

$$V_{Dsat} := V_{GS} - V_T$$

$$I_{spn}(t) := I_0 \cdot \left[1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + \frac{4 \cdot k \cdot T \cdot \gamma}{V_{Dsat}} \cdot \frac{1}{\mu \cdot C_{OX} \cdot \frac{W}{L} \cdot I_0 \cdot \left(1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \right)^2} \right]^2$$

$$I_{spn}(t) := I_0 \cdot \left(1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + \frac{v_n(t)}{V_{Dsat}} \right)^2$$

Expand and drop the vn(t)2 term (noise square is negligible

$$I_{spn}(t) := I_0 \cdot \left(1 + 4 \cdot \frac{A}{V_{Dsat}} \cdot \sin(\pi \cdot f_0 \cdot t) \cdot \cos(\pi \cdot f_0 \cdot t) + 2 \cdot \frac{v_n(t)}{V_{Dsat}} + 4 \cdot \frac{A^2}{V_{Dsat}^2} \cdot \sin(\pi \cdot f_0 \cdot t)^2 \cdot \cos(\pi \cdot f_0 \cdot t)^2 + 4 \cdot \frac{A}{V_{Dsat}^2} \cdot \sin(\pi \cdot f_0 \cdot t) \cdot \cos \right.$$

Trignometric substitution:

$$\sin(\pi \cdot f_0 \cdot t) \cdot \cos(\pi \cdot f_0 \cdot t) = \frac{1}{2} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$ and $$\sin(2 \cdot \pi \cdot f_0 \cdot t)^2 = \frac{1}{2} \cdot (1 - \cos(2 \cdot \pi \cdot 2 \cdot f_0 \cdot t))$$

$$I_{spn}(t) := I_0 \cdot \left[1 + 2 \cdot \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + 2 \cdot \frac{v_n(t)}{V_{Dsat}} + \frac{A^2}{V_{Dsat}^2} \cdot \frac{1}{2} \cdot (1 - \cos(2 \cdot \pi \cdot 2 \cdot f_0 \cdot t)) + 2 \cdot \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \cdot \frac{v_n(t)}{V_{Dsat}} \right]$$

Substitution

$$g_{m0} := \frac{2 \cdot I_0}{V_{Dsat}}$$

$$I_{spn}(t) := I_0 \cdot \left(1 + \frac{A^2}{V_{Dsat}^2} \cdot \frac{1}{2}\right) + \left(g_{m0} \cdot A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)\right) - \frac{I_0 \cdot A^2}{V_{Dsat}^2} \cdot \frac{1}{2} \cdot \cos(2 \cdot \pi \cdot 2 \cdot f_0 \cdot t) \ldots$$

$$+ g_{m0} \cdot v_n(t) + g_{m0} \cdot A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \cdot \frac{v_n(t)}{V_{Dsat}}$$

This expression represents both the large signal currents and the noise currents. The large signal currrent by itself is:

$$I_D(t) := I_0 \cdot \left(1 + \frac{A^2}{V_{Dsat}^2} \cdot \frac{1}{2}\right) + \left(g_{m0} \cdot A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)\right) - \left(\frac{I_0 \cdot A^2}{V_{Dsat}^2} \cdot \frac{1}{2} \cdot \cos(2 \cdot \pi \cdot 2 \cdot f_0 \cdot t)\right)$$

which consists of three components: A DC (or time average) component, the fundamental input component times the small signal transconductance and a second harmonic distortion component. The noise current terms are time varying are represented below

$$I_n(t) := g_{m0} \cdot v_n(t) + g_{m0} \cdot A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \cdot \frac{v_n(t)}{V_{Dsat}}$$

This equation contains two components. The first is a linear cyclostationary white noise component. If viewed under a spectrum analyzer at a rate much less than the input oscillation frequency, the time vary component averages out and the DC component is left

$$g_m(t) := \sqrt{2 \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot I_D(t)}$$

Time varying small signal transconductance

$$g_m(t) := \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_T - A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)\right)$$

$$v_n(t) := 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{g_m(t)}$$

Input referred device thermal noise

$$v_n(t) := 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{\mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_T - A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)\right)}$$

The time average voltage is

$$v_{nave} := 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{\mu \cdot C_{OX} \cdot \frac{W}{L}} \cdot \int_0^{2 \cdot \pi} \frac{1}{\left(V_{GS} - V_T - A \cdot \sin(x)\right)} \, dx$$

$$v_{nave} := \frac{4 \cdot k \cdot T \cdot \gamma}{\mu \cdot C_{OX} \cdot \frac{W}{L}} \cdot 2 \cdot \pi \cdot \frac{\mathrm{csgn}\left[\left(V_{GS} - V_T\right) \cdot \sqrt{\left(V_{GS} - V_T\right)^2 - A^2}\right]}{\sqrt{\left(V_{GS} - V_T\right)^2 - A^2}}$$

$$v_{nave} := \frac{4 \cdot k \cdot T \cdot \gamma}{\mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_T\right)}$$

Or better written as the output referred current noise:

$$i_n(t) := 4 \cdot k \cdot T \cdot \gamma \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_T - A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)\right)$$

The time average of this is

$$i_{nave} := 4 \cdot k \cdot T \cdot \gamma \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot \left(V_{GS} - V_T\right) \qquad v_{nave} = 4 \cdot k \cdot T \cdot \gamma \cdot g_{m0} + \frac{K}{f}$$

which is the same without the large signal input. The last noise term represents a mixing term:

$$\cos(2 \cdot \pi \cdot f_1) \cdot \cos(2 \cdot \pi \cdot f_2) = \frac{1}{2} \cdot \left(\cos(f_1 + f_2) + \cos(f_1 - f_2)\right)$$

2

$$g_{m0}\cdot A\cdot\sin\left(2\cdot\pi\cdot f_0\cdot t\right)\cdot\frac{v_n(t)}{V_{Dsat}}$$

If the noise is white than noise from one location replaces noise from another location and the resultant is a white noise floor. Since noise is uncorrelated from one frequency point to the next it will add uncorrelated to the first component of noise

$$i_{nwhite}(f) := 4\cdot k\cdot T\cdot\gamma\cdot g_{m0}\cdot\left(1+\frac{A}{V_{Dsat}}\right)$$

But, any low frequency noise components such as 1/f noise will noise mix around the carrier. 1/2 will go to one side of the carrier and 1/2 will go to the other side of the carrier

$$i_{n1\_f}(f) := g_{m0}\cdot\frac{A}{V_{Dsat}}\cdot\left[\frac{K}{2\cdot\left(f+f_0\right)}+\frac{K}{2\cdot\left(f-f_0\right)}\right]$$

The net result is:

1. The output current increases by $(1+A^2/V_{Dsat}{}^2/2)$
2. The output white current noise increases by (1+A/VDsat)
3. The output 1/f noise is simply g_{m0}*K/f
4. Two new 1/f terms are added: g_{m0}*A/V_{Dsat}*K/2*(1/(f-f_0)+1/(f-f_0))
5. The fundamental exists: g_{m0}*v_i
6. A second harmonic existis: g_{m0}*A/V_{Dsat}/2*v_i
